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OSCILLATORY MOTION OF LIQUID IN COMPOSITE CAPILLARIES

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Abstract. The paper presents an analysis of the dynamic behavior of oscillatory flows in elements of capillary systems based on previously conducted studies. Considered configurations include two capillaries of different diameters connected in series, as well as a pipe with two branching capillaries (a branching system element). Such elements are typical for hydraulic and capillary structures found in both technical and biological systems. The purpose of this research was to extend the previously obtained results to more complex cases involving sharp changes in the cross-section of capillaries and their branching. In this case, composite capillaries were understood as systems of connected capillary pipes with different lengths and diameters.

The study begins by applying the theory of laminar flow and established approaches from the literature to determine flow rates through a capillary of constant diameter under oscillatory inlet conditions. Next, the phase shift between the oscillations of flow rate and pressure within the capillary is calculated as a function of the capillary radius. It was found that the phase shift decreases with decreasing capillary radius, but increases with increasing oscillation frequency. Using mass conservation laws, the phase shifts of flow and pressure oscillations in different sections of a composite linear capillary are then derived. It is shown that the phase shift of flow rate relative to pressure oscillation depends on the diameters of the two sections. The phase shift of pressure also varies relative to the first capillary, but the sum of the phase shifts of pressure and flow rate in each section remains constant. Further analysis of a branching system element leads to a general and simple rule: the sum of phase shifts in each branch of a capillary junction remains constant.

Thus, the previously established patterns for capillaries with weakly varying diameters are also valid for more complex composite capillary systems — those with sharp changes in diameter and branching geometries. This result may be useful for evaluating the distribution of total flow within complex branching capillary networks.

Keywords: capillary, liquid, flow rate, mass transfer, oscillations

1. Introduction

The oscillatory nature of fluid motion in porous and capillary systems accompanies many natural and technological processes. In the mining industry, it is directly related to the state of rock masses, as well as to oil and natural gas extraction technologies [1], technologies for the utilization of secondary resources, and the preparation of mineral raw materials for further processing. Oscillatory motion also plays a significant role in biological systems, particularly in capillary processes in plants and in the circulatory systems of animals and humans [2, 3]. In many cases, the study of flows in porous media requires investigation of mass transfer processes at the pore scale, e.g., [4]. In most situations, to describe the process in greater detail, the problem is reduced to the study of capillary flows in individual tubes—i.e., a topologically complex porous system is simplified to a certain network of capillaries [5]. This simplified model makes it possible to address rather complex physical problems and to provide qualitative recommendations for understanding processes in real-world scenarios. One of the main directions associated with the dynamic behavior of such systems and with heat and mass transfer processes involves the study of unsteady, and in particular, pulsating fluid flows.

The range of applications for such problems is currently quite broad (described, for example, in reviews [6–8]). The first of these publications [6] highlights the virtually limitless prospects in the field of microfluidics related to generating pulsatile oscillations in microcapillaries, mixing solutions, targeted drug delivery within capillary systems, and more. In the other two publications [7, 8], the authors,

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analyzing the current state of understanding of processes in the cardiovascular system, emphasize the need for extensive use of mathematical models in this area. Regarding unsteady and pulsatile flows specifically, classical formulations of these problems are presented in [9, 10].

Recently, a large number of studies have been conducted, for example, [11–15], mainly focused on modeling the dynamics of blood flow in the circulatory system. In [11, 12], the problem of oscillatory flows in capillaries is examined based on classical solutions. The first study, neglecting inertial terms, obtains solutions for flows with oscillating capillary walls. The second study provides an analytical solution for a planar case involving a mixture of fluids, one Newtonian and the other Maxwellian.

The next group of studies [13–15] considers more complex models of blood flow in capillaries considering the surrounding tissues, where a filtration flow model is assumed. In [13], the fluid in the capillary is treated as non-Newtonian due to the influence of hematocrit, while in the latter studies, pressure pulsations are described as piecewise linear segments. All these publications highlight the significant influence of pressure pulsations

An interesting practical approach related to pulsatile blood flow is presented in [16]. Using wavelet coherence analysis, a relationship between oscillations in the venous and arterial parts of the capillary and their phase difference is established, which is directly relevant to the present work.

This research, based on the aforementioned approaches, investigates oscillatory fluid motion in composite capillary systems. The current study is an advancement of the work [17], which investigated certain features of oscillatory motion in a capillary with a slightly varying radius.

The aim of this research is to extend previously obtained results to more complex cases involving sharp changes in the capillary cross-section and branching. Composite systems are determined here as networks of interconnected capillary tubes with varying lengths and diameters.

2. Methods

Below, the main equations governing unsteady motion in a capillary tube of constant diameter are presented:

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\rho \partial z} + \nu \frac{\partial}{r \partial r}\left(r \frac{\partial u}{\partial r} \right), \tag{1}$$

$$\frac{\partial (ru)}{\partial z} + \frac{\partial (rw)}{\partial r} = 0, \tag{2}$$

where t is time; z is the longitudinal coordinate along the capillary axis; r is the radius; u – longitudinal velocity; w – the transverse velocity, which in this study equals zero; ρ – density; p – pressure; and ν – kinematic viscosity coefficient. For pulsatile flow, the problem has an analytical solution presented in [9]. It is expressed as a zeroth-order Bessel function with a complex argument. In our case, to obtain the required parameters, it is convenient to use a numerical solution directly. According to [9], the pressure gradient and velocity can be represented as $\frac{\partial p}{\rho \partial z} = \sin(2\pi f t)\frac{dP}{\rho dz}$, $u = \sin(2\pi f t)U_s + \cos(2\pi f t)U_c$, where *f* is the oscillation frequency. Then, for the functions U_s, U_c, the following equations are

$$\frac{d^2 U_s}{dn^2} + \frac{dU_s}{ndn} + \chi U_c = \frac{R_c^2}{\nu}\frac{dP}{\rho dz}\ , \tag{3}$$

$$\frac{d^2 U_c}{dn^2} + \frac{dU_c}{ndn} - \chi U_s = 0, \tag{4}$$

with the boundary conditions:

for n=0

$$\frac{dU_s}{dn} = \frac{dU_c}{dn} = 0, \tag{5}$$

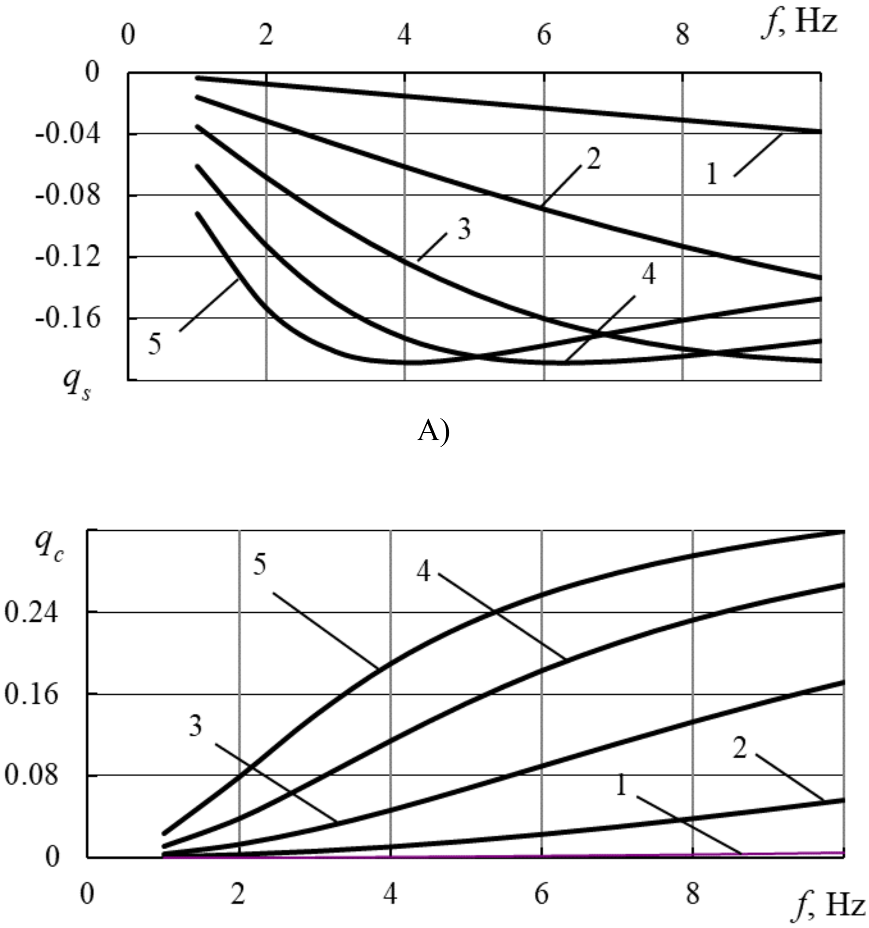
for n=1

$$U_S = U_C = 0. \tag{6}$$

In these equations $\chi = 2\pi f R_c^2/\nu$, P are the amplitudes of pressure oscillations, R_c is the capillary radius and $n=r/R_c$.

The procedure of the numerical solution is as follows. The solution is represented in the form: $U_s = \frac{R_c^2}{\nu\chi}\frac{dP}{\rho dz}V_s$, $V_s = B_s$ and $U_c = \frac{R_c^2}{\nu\chi}\frac{dP}{\rho dz}V_c$, $V_c = 1 + B_c$, where the functions φ_s and φ_c satisfy the homogeneous equations (3) and (4). We expand these functions into series as $\varphi_s = 1 + As_2 n^2 + As_4 n^4 + As_6 n^6 + ...$ and $\varphi_c = Ac_0 + Ac_2 n^2 + Ac_4 n^4 + Ac_6 n^6 + ...$ After substituting these expressions into equations (3) and (4), the obtained coefficients are expressed in terms of Ac_0. Then, starting the integration with a small initial step from zero point, we adjust the coefficient Ac_0 so that at n = 1, φ_s = 0. After that, taking B = - 1/φ_c (1), the solutions for V_s and V_c is obtained, as well as for the flow rate $Q = Q_s\sin(2\pi f t) + Q_c\cos(2\pi f t)$, where $Q_s = \frac{R_c^4}{2\pi f}\frac{dP}{\rho dz}q_s$, $q_s = \int_0^1 nV_s dn$ and $Q_c = \frac{R_c^4}{2\pi f}\frac{dP}{\rho dz}q_c$, $q_c = \int_0^1 nV_c dn$.

Figure 1 shows the curves of dimensionless flow rates as a function of frequency for different capillary radii from 0.1 mm to 0.5 mm. Considering that equations (3) and (4) contain only one parameter, χ, all the necessary dependencies can be constructed based on its value. The presence of the q_c and q_s values indicates that the fluid flow oscillates with a phase shift δ ($tg\delta = Q_c/Q_s = q_c/q_s$) relative to the pressure gradient oscillation. The magnitude of this phase shift, as it is mentioned above, depends on the parameter χ (Figure 2). It follows from this figure that as the parameter χ decreases, the phase shift tends to zero. This means that either a decrease in frequency or a decrease in capillary radius leads to a reduction in the phase shift. On the other hand, even for very narrow capillaries, a significant phase shift can be achieved by increasing the oscillation frequency.



B)

1 – R_c = 0.1; 2 – R_c = 0.2; 3 – R_c = 0.3; 4 – R_c = 0.4; 5 – R_c = 0.5 mm

Figure 1 – Dimensionless flow rates q_c (A) and q_s (B) as a function of oscillation frequency for different capillary radii

Now, using the obtained solutions, it is possible to construct the pulsation dynamics for composite and branched capillaries. Let us consider it using two elementary types of composite capillaries as examples.

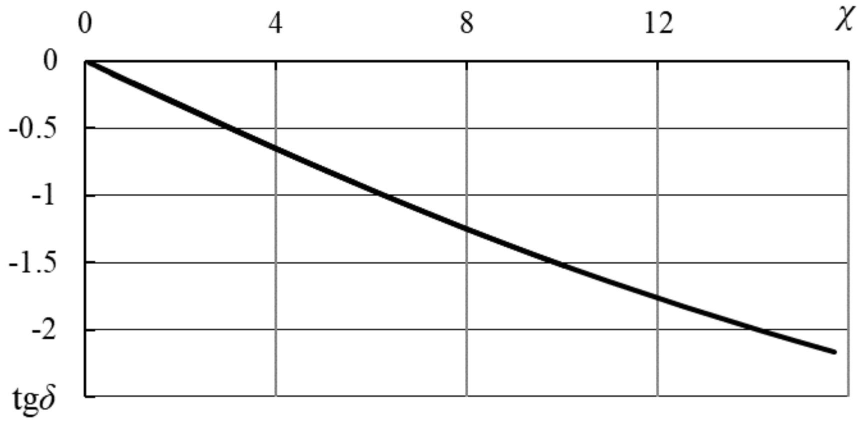


Figure 2 – Variation of tgδ as a function of parameter χ

3. Results and discussion

Composite linear capillaries. By composite linear capillaries, we mean capillaries consisting of sequentially connected long segments with different, but constant within each segment, diameters. We assume that the pressure drops arising at the transitions between segments are much smaller than the resistance of an individual capillary segment. In this case, the total resistance can be considered as the sum of the resistances of all segments. The pressure gradient for the *j*-th segment of the composite capillary can be written in the general form for harmonic functions as follows:

$$\frac{dP_j}{\rho dz} = P_j \sin\left[2\pi f\left(t+\alpha_j\right)\right] = P_j\left[\sin(2\pi ft)\cos\left(2\pi f\alpha_j\right)+\cos(2\pi ft)\sin\left(2\pi f\alpha_j\right)\right], \qquad (7)$$

where α_j defines the phase of the harmonic oscillation. For a single capillary, it depends only on the choice of the initial point on the time axis. In the expression (7), it was assumed that the pressure gradient in the *j*-th segment is determined solely by the geometrical characteristics of that particular channel. It was also taken into account that the transition from one capillary to another is associated with a sharp change in radius, which may lead to a change in the phase angle. Therefore, in composite capillaries, the phases of adjacent segments must be related to each other. From the form of equations (3) and (4), it follows that the oscillating fluid flow rate Q differs in shape from the pressure gradient due to a phase shift. Thus, the flow rate in the *j*-th segment of the capillary can be written as:

$$Q_j = G_j \sin\left[2\pi f\left(t+\alpha_j+\delta_j\right)\right] =$$
$$= G_j\left\{\cos\left[2\pi f\left(\alpha_j+\delta_j\right)\right]\cdot\sin(2\pi ft)+\sin\left[2\pi f\left(\alpha_j+\delta_j\right)\right]\cdot c\right.$$
$$= G_j\left\{\begin{array}{l}\sin(2\pi ft)\left[\cos\left(2\pi f\delta_j\right)\cos\left(2\pi f\alpha_j\right)-\sin\left(2\pi f\delta_j\right)\sin(2\right.\\ +\cos(2\pi ft)\left[\sin\left(2\pi f\delta_j\right)\cos\left(2\pi f\alpha_j\right)+\cos\left(2\pi f\delta_j\right)\sin\right.\end{array}\right.$$

, (8)

where $G_j = \frac{R_{cj}^4}{2\pi f} P_j \left(q_{sj}^2 + q_{cj}^2\right)^{1/2}$ is uniquely related to P_j, and δ_j defines the phase difference between the flow oscillation and the pressure gradient oscillation. Considering that if the oscillation frequency and the radii of the channels are known, then the phase deviations δ_j in each segment are also known.

Let us consider a specific case of a composite capillary consisting of two segments with lengths l_1 and l_2. In this case, the total pressure drop at the ends of the capillary will be equal to:

$$\Delta p = \rho \begin{Bmatrix} [P_1 l_1 \cos(2\pi f \alpha_1) + P_2 l_2 \cos(2\pi f \alpha_2)]\sin(2\pi f t) + \\ + [P_1 l_1 \sin(2\pi f \alpha_1) + P_2 l_2 \sin(2\pi f \alpha_2)]\cos(2\pi f t) \end{Bmatrix}. \quad (9)$$

At the interface between the segments, the condition of flow rate continuity must be satisfied, since the flow rate is a continuous function and does not depend on the segment number.

$$G_1 \cos[2\pi f(\alpha_1 + \delta_1)] = G_2 \cos[2\pi f(\alpha_2 + \delta_2)], \quad (10)$$

$$G_1 \sin[2\pi f(\alpha_1 + \delta_1)] = G_2 \sin[2\pi f(\alpha_2 + \delta_2)]. \quad (11)$$

These conditions reveal an interesting aspect. Suppose the fluid velocity in the first capillary is described by a sine law; then, in the second capillary, the flow must also oscillate according to the same law. From equation (11), it follows that this is possible if $\sin[2\pi f(\alpha_1 + \delta_1)] = 0$ and $\sin[2\pi f(\alpha_2 + \delta_2)] = 0$, i.e. $\alpha_2 + \delta_2 = \alpha_1 + \delta_1 = 0$. This relation defines the oscillation phases of the pressures in the first and second segments (one of the phases can be chosen arbitrarily). By substituting these values into equation (9), the variation of the total pressure at the ends of such a tube can be determined.

Alternatively, the problem may be reformulated by specifying the total pressure drop at the tube ends, for instance, according to the same sine function, i.e.

$$P_1 l_1 \cos(2\pi f \alpha_1) + P_2 l_2 \cos(2\pi f \alpha_2) = \Delta P, \quad (12)$$

under these conditions

$$P_1 l_1 \sin(2\pi f \alpha_1) + P_2 l_2 \sin(2\pi f \alpha_2) = 0, \quad (13)$$

In this case, $\alpha_1 = 0$ and $\alpha_2 = 0$ cannot be assumed, as this would lead to the following simplifications in formulas (10) and (11) $G_1 \cos(2\pi f \delta_1) = G_2 \cos(2\pi f \delta_2)$, $G_1 \sin(2\pi f \delta_1) = G_2 \sin(2\pi f \delta_2)$.

From the last two expressions, it follows that condition $\mathrm{tg}(2\pi f \delta_1) = \mathrm{tg}(2\pi f \delta_2)$ is satisfied only when the radii of the first and second channels are equal, which contradicts the assumptions of the problem.

From condition (13), it follows that despite the sinusoidal nature of the total pressure drop oscillation, additional pressure differences arise within each tube. These vary according to a cosine law and have opposite signs, so that their sum equals zero. In this case, dividing equation (11) by equation (10), the following expression is obtained $\mathrm{tg}[2\pi f(\delta_1+\alpha_1)] = \mathrm{tg}[2\pi f(\delta_2+\alpha_2)]$ or

$$\alpha_2 + \delta_2 = \alpha_1 + \delta_1 , \tag{14}$$

considering that $G_2 = G_1$,

$$R_{c_1}^4 P_1 \left(q_{s_1}^2 + q_{c_1}^2\right)^{1/2} R_{c_2}^4 P_2 \left(q_{s_2}^2 + q_{c_2}^2\right)^{1/2} . \tag{15}$$

From (12) and (15), it is found that

$$P_1\left[l_1 + \left(\frac{R_{c_1}}{R_{c_2}}\right)^4 \frac{\left(q_{s_1}^2 + q_{c_1}^2\right)^{1/2}}{\left(q_{s_2}^2 + q_{c_2}^2\right)^{1/2}} l_2\right] = \Delta P , \tag{16}$$

using (13) and considering (14), the following result is obtained:

$$\mathrm{tg}\,\alpha_1 = -\frac{P_2 l_2}{P_1 l_1 + P_2 l_2 \cos(\delta_1 - \delta_2)} \sin(\delta_1 - \delta_2), \tag{17}$$

From this, it indeed follows that α_1 and, accordingly, α_2 become equal to zero when $\delta_1 = \delta_2$. Thus, condition (14) establishes the relationship between the phases of pressure oscillations in two adjacent segments. The study [16] demonstrated that blood flow oscillations within the cardiac, respiratory, and myogenic frequency ranges can exhibit high and reliable phase coherence across different parts of the capillary. In the specific investigated case, blood flow velocities were found to be in phase, i.e., condition (14) is satisfied. The authors of the mentioned article suggest that disruptions in phase coherence and stability of velocity relationships may indicate certain vascular changes. Thus, this simple condition may be necessary for analyzing the state of the cardiovascular system.

Branched capillaries. Samples of branched channels are widely represented in technical and biological systems [18–20]. For elements of a branched capillary, assuming equal pressure drops at the branching points, the following relationships apply:

$$\begin{aligned} P_2 l_2 \cos(2\pi f \alpha_2) &= P_3 l_3 \cos(2\pi f \alpha_3), \\ P_2 l_2 \sin(2\pi f \alpha_2) &= P_3 l_3 \sin(2\pi f \alpha_3). \end{aligned} \tag{18}$$

Indices 2 and 3 denote the two branches of the capillary. From relations (18), it follows that

$$\alpha_2 = \alpha_3 ,\qquad P_2 l_2 = P_3 l_3 , \tag{19}$$

That is, the pressure in the branches oscillates in the same phase. From equation (19), it follows that

$$\frac{l_2}{R_{c_2}^4} \frac{G_2}{\left(q_{s_2}^2 + q_{c_2}^2\right)^{1/2}} = \frac{l_3}{R_{c_3}^4} \frac{G_3}{\left(q_{s_3}^2 + q_{c_3}^2\right)^{1/2}} , \tag{20}$$

$$\begin{aligned} &G_1 \left[\cos(2\pi f\delta_1)\cos(2\pi f\alpha_1) - \sin(2\pi f\delta_1)\sin(2\pi f\alpha_1)\right] = \\ &\left[G_2 \cos(2\pi f\delta_2) + G_3 \cos(2\pi f\delta_3)\right]\cos(2\pi f\alpha_2) - \\ &- \left[G_2 \sin(2\pi f\delta_2) + G_3 \sin(2\pi f\delta_3)\right]\sin(2\pi f\alpha_2) \end{aligned} , \tag{21}$$

$$\begin{aligned} &G_1 \left[\sin(2\pi f\delta_1)\cos(2\pi f\alpha_1) + \cos(2\pi f\delta_1)\sin(2\pi f\alpha_1)\right] = \\ &\left[G_2 \sin(2\pi f\delta_2) + G_3 \sin(2\pi f\delta_3)\right]\cos(2\pi f\alpha_2) + \\ &+ \left[G_2 \cos(2\pi f\delta_2) + G_3 \cos(2\pi f\delta_3)\right]\sin(2\pi f\alpha_2) \end{aligned} . \tag{22}$$

In these expressions, index 1 denotes the segment before the branching point. By squaring both the left- and right-hand sides of equations (21) and (22), and then summing them accordingly, the following is obtained:

$$G_1^2 = G_2^2 + 2\cos\left[2\pi f(\delta_2 - \delta_3)\right]G_2 G_3 + G_3^2 , \tag{23}$$

It follows that the condition $G_1 = G_2 + G_3$ is satisfied only when $\delta_1 = \delta_2$. Taking into account equation (20), expression (23) yields:

$$\left\{1 + 2\frac{l_2}{l_3}\left(\frac{R_{c_3}}{R_{c_2}}\right)^4 \frac{\left(q_{s_3}^2 + q_{c_3}^2\right)^{1/2}}{\left(q_{s_2}^2 + q_{c_2}^2\right)^{1/2}} \cos\left[2\pi f(\delta_2 - \delta_3)\right] + \left[\frac{l_2}{l_3}\left(\frac{R_{c_3}}{R_{c_2}}\right)^4\right]^2 \frac{\left(q_{s_3}^2 + q_{c_3}^2\right)^{1/2}}{\left(q_{s_2}^2 + q_{c_2}^2\right)^{1/2}}\right\}^{1/2} G_2 = G_1 , \tag{24}$$

By eliminating G_1 from conditions (21) and (22), it is obtained that $G_2 \cdot \sin(\alpha_1 + \delta_1 - \alpha_2 - \delta_2) + G_3 \cdot \sin(\alpha_1 + \delta_1 - \alpha_2 - \delta_3) = 0$ or

$$\begin{aligned} &\left[\cos(\alpha_1 + \delta_1 - \delta_2) + \cos(\alpha_1 + \delta_1 - \delta_3)\frac{G_3}{G_2}\right]\mathrm{tg}\,\alpha_2 = \\ &\left[\sin(\alpha_1 + \delta_1 - \delta_2) + \sin(\alpha_1 + \delta_1 - \delta_3)\frac{G_3}{G_2}\right] \end{aligned} . \tag{25}$$

From (25), taking into account (20), the phase of the pressure oscillation in the branched part relative to the oscillation in the first segment of the composite capillary can be obtained. If the diameters of tubes in segments 2 and 3 are equal, then $\delta_3 = \delta_2$ and $\mathrm{tg}\,\alpha_2 = \mathrm{tg}(\alpha_1 + \delta_1 - \delta_2)$ or $\alpha_2 + \delta_2 = \alpha_1 + \delta_1$, for any ratio of G_3 / G_2. This result coincides with that for the linear composite capillary (equation (14)). Thus, considering condition (23), it can be stated that these two channels are equivalent to a single one with a total flow rate G_1. In the general case, if it is initially assumed that $\alpha_2 + \delta_2 = \alpha_3 + \delta_3$ and the conditions (21) and (22) are rearranged, the equality $G_1 = G_2 + G_3$ can be obtained, which holds for different diameters of the channels α_2 and α_3. Therefore, at the channel branching, oscillations in the different branches exhibit some phase shift, while the sum $\alpha_j + \delta_j$ (j = 1, 2, 3) remains constant for all elements.

4. Conclusion

The analysis of the study results suggests that the total phase shift, in cases where there are abrupt changes in the cross-sectional areas of the capillaries and even when the topological structure of the channel changes sharply, remains constant for each segment of the channel. This conclusion is consistent with the authors' earlier findings for channels with slightly varying radii [17].

The obtained results may have practical applications in the study of branched capillary systems and hydraulic distribution lines.

Conflict of interest

Authors state no conflict of interest.

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КОЛИВАЛЬНИЙ РУХ РІДИНИ В СКЛАДЕНИХ КАПІЛЯРАХ

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Анотація. У цій роботі на основі виконаних раніше досліджень розглянуто динамічні особливості коливальних рухів в елементах капілярних систем. Такими елементами, наприклад, є послідовно пов'язані два капіляри з різними діаметрами, або трубочка з двома капілярами, що відходять з неї (елемент розгалуженої системи). Подібні елементи поширені у будь-яких гідравлічних та капілярних утвореннях, що відносяться як до технічних, так і до біологічних систем. На початку роботи на основі теорії ламінарного руху та відомих з літератури підходів показано чисельні значення витрат через капіляр постійного діаметра при накладанні коливань на вході. Потім визначаються і наводяться значення фазового зміщення коливань витрат щодо коливань тиску в капілярі в залежності від його радіуса. Далі на основі законів про збереження мас встановлюються залежності фазових зсувів коливань витрат і тисків в різних частинах складеного лінійного капіляра. Було встановлено, що, в залежності від діаметрів двох частин складеного капіляра змінюються фазові зміщення витрат щодо коливання тиску. Змінюються також фазове зміщення тисків щодо першого капіляра, але при цьому сума зміщень коливань тиску і витрати в кожному капілярі залишається постійною. Подальший аналіз, проведений для елемента розгалуженої системи, призводить до загальної простої умови, що сума фазових зсувів для кожної частини розгалуженого елемента капілярної системи залишається величиною постійною.

Таким чином, отримані раніше закономірності для капілярів зі слабо змінним діаметром, залишаються справедливими для більш складних складових капілярних систем, під якими в даному випадку розуміються капіляри з різкими змінами діаметра і капіляри, що розгалужуються. Цей результат, можливо, буде корисний для оцінок розподілу загальної витрати в елементах складних капілярних систем, що гілкуються.

Ключові слова: капіляр, рідина, витрата, масообмін, коливання.